Optimal Storage Investment and Management under Uncertainty

It is costly to avoid outages!

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Abstract—Subject of this analysis is to show how storage is operated optimally under renewable and load uncertainty in the electricity system context. We estimate a homogeneous Markov Chain representation of the residual load in Germany in 2014 on an hourly basis and design a very simple dynamic stochastic electricity system model with non-intermittent generation technologies and storage. We compare these results to perfect foresight findings and identify a significant over estimation of the storage potential under perfect foresight.

Keywords—storage, electricity system, uncertainty

Categories—electricity and nuclear, energy security, energy modelling, energy economics

1. Overview

In electrical systems storage has the technical potential to increase efficiency significantly - especially in the context of integrating intermittent renewable technologies. This is achieved by shifting energy from periods of low demand to periods of high demand, which raises the utilization of base load power plants and reduces that of peak load power plants. The full gain is achieved if generation capacity is adapted to the "equilibrated" load situation - with a higher base load capacity and fewer peaking stations. In this case, the installed fossil generation capacity might fall below peak load level. Since the amount of energy stored is limited, there is a risk of expensive outages in cases of prolonged demand peaks.

Many previous analyses of storage are based on perfect foresight models in which the operator could ensure that the store always approaches a prolonged peak with just enough energy to avoid an outage. In the real world, it may be impossible to predict the length of a peak, and a different strategy is needed: taking this issue into account, our aim is to derive the optimal way of integrating the storage into the system.

We estimate a homogeneous Markov Chain representation of the residual load in Germany in 2014 on an hourly basis (section 2.1) and design (section 2.2) a very simple dynamic stochastic electricity system model with fossil generation technologies and storage (a Markov Decision Process). This model is solved in section 3 for a stationary state using numerical methods (linear optimization) and the optimal storage strategy is presented. It is shown that under uncertainty at high demand an increasing share of the storage is "frozen" in its charged state to avoid lost load (outages). Therefore a "buffer share" of the storage is not used for equilibration of load any more. Furthermore, this buffer state of charge is established, if necessary, even in periods of high demand, so that the storage operation stresses the system.

To implement the full efficiency potential of storage, generating capacities have to be reduced. Peak load can then exceed installed capacity. If this is the case, in the optimum under load uncertainty a storage buffer is created and maintained. In section 4 the optimal strategy is compared to the optimal

solution derived under perfect foresight of explicit drawings of the stochastic load process. It is shown that a "buffering" is not required in optimal storage operation under perfect foresight assumption. Furthermore compared to the perfect foresight equivalent, the more realistic storage management strategy includes more "waiting". As result, the storage cannot be operated as efficiently as in the perfect foresight case, reducing the cost savings available.

We also find that an increasing risk of reaching peak load further reduces the efficiency potential of the storage. Since the optimal storage strategy is not implemented "naturally" by competitive storage operators, it might be advisable not to adjust generation fully in response to the growth of storage, reducing the difficulty of regulating it. This analysis refines the assessment of the economic potential of electricity storage, thus contributing to more effective planning of energy systems of the future, where outages are avoided.

2. Methodology

We derive the expected cost minimizing way of operating energy storage and non-intermittent generation and adjusting non-intermittent capacities for a given storage capacity (300 GWh in our case study). The operator aims to satisfy demand while processing sequentially revealed information about the uncertain residual load. The problem is stochastic and multiscale as it includes short term information processing, storage management and generation decisions as well as long term investment decisions in generation capacities. We develop a dynamic stochastic electricity system optimization model as a Markov Decision Process. A solution is an optimal strategy that assigns each state - defined by the amount of stored energy, residual demand and non-intermittent generation capacities - a probability distribution over possible charging and discharging values. The nonintermittent generators run in merit order to meet the residual demand plus charging (or minus discharging). The model is quantified with an estimated homogeneous Markov Chain representation of the residual load (demand minus wind and solar output) in Germany in 2014 on an hourly basis and with technology cost data. The model is solved for a stationary policy using a linear optimization approach embedded in a hill climbing capacity optimization environment. This strategy and the stationary probabilities are analysed using counter factual experiments and they are compared to the optimal solution derived under perfect foresight of explicit drawings of the stochastic load process. Thus features of the optimal strategy can be derived and the perfect foresight "error" can be quantified.

2.1 Modelling residual load in Germany as Markov Chain

In this section it is examined to what extent the modelling of the residual load of Germany in 2014 in hourly resolution a) as a homogeneous Markov Chain and b) in 10 GW steps is empirically valid¹. The residual load is defined as load reduced by renewable generation - in the case of Germany, mainly wind and solar power.

		Load	Wind	Solar	Residual Load	
Total	GWh	504166	51443	32816	419906	
Day Mean	GWh	1381	141	90	1150	
Max	GW	79	29	24	78	
Min	GW	35	0	0	14	
Table 1: Benchmarks of residual load in Germany						

The annual load in Germany amounts to 500 TWh. 10% of this load are generated by wind and 7% by solar. Herewith the residual load is reduced to 83%. The peak load is 79 GW. At maximum 29 GW of

¹ See [13] and [14] for a discussion of Markov Load modelling.

wind and 24 GW solar respectively are produced. Nevertheless, the peak of the residual load remains at 78 GW, which reflects the intermittent character of renewable generation. The load will never fall below 35 GW; the minimum of the residual load, however, is only half of the lower bound (14 GW). It is obvious that renewables a) cover a substantial share of the load (17%), b) halve the minimum load but c) do not decrease peak load (the residual load duration curve is shown in figure 1, red).





Figure 1: Load Duration: German data 2014 (red curve), rounded German data 2014 (black step function), stationary distribution of the Markov Chain (blue bar step function)

Figure 2: Partial autocorrelation plot of the German residual load 2014 (orange) and of a simulated path of the Markov Chain

The (consistent) maximum likelihood estimators of the transition matrix of a homogeneous Markov Chain are the numbers of state transitions normalized per line. The estimator of the transition matrix for the residual load in Germany is²:

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.02 & 0.79 & 0.19 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.03 & 0.8 & 0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.05 & 0.81 & 0.14 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.12 & 0.76 & 0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.22 & 0.73 & 0.07 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.22 & 0.77 & 0.01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.45 & 0.55 \end{pmatrix}$$

The estimated Markov Chain is a) "irreducible", thus all states are mutually accessible, b) "aperiodic", i.e. the greatest common divisor of the return times to the initial states is one and c) all states are "positive recurrent", i.e. with probability 1 there is a return to the initial state and the anticipated return time is finite. In this case, there is a steady state distribution q with $\lim_{t\to\infty} p(t) = \lim_{t\to\infty} p(0)P^t = q$, independent of the initial distribution. q is the solution of the linear system of equations q = P q, q1 = 1. The solution of this system for the estimated Markov Chain P results in the stationary distribution: q [%] = {<1, 1, 9, 29, 34, 20, 6, <1}.

If these probabilities are interpreted as interval length and the states are ordered, q can be interpreted as load duration. Compared with the original data, the approximation of the discretized load (figure 1, black step function) by the Markov Chain (blue step function) is accurate.

Intuitively the estimated Markov Chain is a sound representation of the frequency of the state transitions of the residual load. It is therefore not surprising that also the long-term distribution of the residual load is well approximated. However, periodic structures, such as daily, weekly and annual rhythms cannot be approximated by aperiodic first order Markov Chains. Thus, the partial autocorrelation coefficients of the residual load (figure 2) are significant (5% \pm 0.02) for almost all lags

² P is the matrix of probabilities p_{ij} , p_{ij} is the probability that residual demand of size 10xi GW is followed by residual demand 10xj GW. p_{11} is the element in the upper left corner.

(orange points). In contrast, the coefficients of the Markov Process are only significant for a lag of one hour

As an illustration of the two processes, respectively the difference between them, in figure 3: a) a 500 hours segment of the residual load in Germany, b) a 500 hours segment of the rounded (to 10 GW) residual load in Germany and c) a 500 hours simulation of the estimated Markov Chain is shown. It is not obvious that there is a fundamental difference between time series b) and c).

Figure 3: a) 500 hours segment of the residual load in Germany 2014, . b) 500 hours segment of the rounded residual load in Germany 2014, c) 500 hours simulation of the estimated Markov Chain

In summary, the modelling of the residual load as a first order homogeneous Markov Chain represents the load transitions and the load duration very well. With this interpretation, a stochastic energy system model is formulated in section 2.2. Nevertheless, the order of load changes is random and does not reflect the natural daily, weekly and seasonal rhythms. Therefore, there is substantially more uncertainty about the future residual load development represented in the model than in reality. Thus the Markov approach to residual load modelling would be more appropriate for a scenario with a higher share of intermittent renewables. The order of the load transitions is relevant for the operation of the storage.

2.2 Electricity System Model

In the following welfare-maximizing capacity-, output- and storage-decisions are determined to derive the social value of electricity storage. Welfare is interpreted as system cost. In the following case study the availability of a free storage capacity $\hat{S} = 300$ GWh (equivalent to 6 hours average load in Germany; for an overview of storage potential in Europe see [15]) is assumed. It is then possible to compare the system cost in a scenario with storage to a scenario without storage and thus to determine the value of storage in terms of system cost avoidance. This comparison is on the one hand conducted under perfect foresight (Det) of the residual load D_t [GW] and on the other hand under stochastic residual load (Sto), as described in the previous chapter, to identify the impact of the two assumptions on the valuation of a storage option.

Electricity can be generated by a portfolio of non-intermittent technologies - modelled as simple stack in contrast to generator stack. The vector $x_t \ge 0$ [GW] describes the related production per hour. To apply these generation technologies initial investments with capacities k [GW] have to be made (green field approach). The capacity limits the non-intermittent generation $k \ge x_t$. s_t [GW] corresponds to the charging or discharging of the storage per hour. The stored energy is S_t [GWh] ($\hat{S} \ge S_t \ge 0$). There are neither politically motivated interventions (CO₂ prices) nor technical restrictions that impact the use of specific technologies - neither distribution effects are taken into account (copper plate assumption). The alternatives are evaluated in a system cost approach that includes fixed costs $c^{var} k$ and variable costs $c^{var} x_t$ over 40 years. In the deterministic as well as in the stochastic model the time resolution is one hour. In the stochastic model only a single hour is modelled and then extrapolated to 40 years, while in the deterministic model the result of a year is extrapolated. Costs are not discounted but modelled as long-term variable average costs in ϵ /hour³.

The monetary value of unsatisfied demand (*VoLL*, value of lost load) has been quantified in comprehensive empirical studies. Following [11], the social planner can decide upon capacity and production such that - assuming the unsatisfied demand is valued with this *VoLL* = 100 max $\{c^{var}\}^4 - a$ satisfaction of the complete demand is discarded in favour of lower system costs. The satisfaction of the residual load is therefore not modelled as a restriction, but incorporated as part of the objective function⁵. This has the particular advantage that an endogenous determination of reserve capacity is possible within the model. The latter is described as desirable by [18] for future electricity system modelling approaches that are able to consider and to evaluate the effects of the inclusion of additional intermittent renewables quantitatively.

Variable and fixed costs of non-intermittent generation technologies based on [20] are presented in table 2:

Technology		Coal	IGCC	Combus turbine	t Combine cycle	d Nuclear	
Variable Cost (Fuel+OM)	€/MWh	27	25	55	40	22	
Fix cost	€/KW	2000	2500	650	800	3250	
Table 2. tool	analami	coocific or	act data.	constant	lifetime of	the plants of	. f

Table 2: technology specific cost data; constant lifetime of the plants of 40x365x24[h/plant], Source: [20]

The welfare-maximizing capacity-, output- and storage-decisions under the perfect foresight assumption are the solution of the following problem with T = 365×24 and $\eta_{Det} = 40 \times T$:

$$\min_{x_t,k,s_t} c^{fix}k + \frac{\mu_{Det}}{T} \sum_{t=1}^{T} c^{var} x_t + VoLL y_t \tag{1}$$

$$y_t \ge D_t + s_t - x_t \tag{2}$$

s.t.:
$$\hat{S} \ge S_t = S_{t-1} + s_t \ge 0$$
 (3)

$$s.t.: k \ge x_t \ge 0 \tag{4}$$

$$s.t.: y_t \ge 0 \tag{5}$$

In contrast to the perfect foresight model in the case of uncertainty the information available about the residual load will be considered as well as the impact of output and storage decisions on the future revenues. These requirements are met in the stochastic dynamic programming approach by the

⁴ For a literature review see [7], [8], [9], [10].

³ To give up discounting avoids implausible unloading in storage models. Similarly Tijms [6] explains: "For many applications of Markov decision theory this criterion is the most appropriate optimality criterion. The average cost criterion is particularly appropriate when many state transitions occur in a relatively short time."

⁵ Modeling the *VoLL* reminds of the "penalty"-approach to the numerical solution of constrained nonlinear optimization problems. This analogy can be used to apply an (intuitive) theorem that sheds light on the modelling approach: The penalty solution converges to the solution of the constrained problem, as *VoLL* tends to infinity. Proof: penalty methods e.g. Luenberger (1984).

determination of an optimal strategy (or "policy"⁶, which consists of decision rules for each state). Every decision rule allows to select an action on the basis of the occurring state from a distribution of alternatives (actions). In our case, the state of the system consists of the residual load D_t , the energy stored S_t and the initially fixed non-intermittent generation capacities k and the storage capacity \hat{S} . The decisions (actions) in each period are the level of non-intermittent generation x_t and the change of the state of charge s_t ?. A decision rule f_t assigns a distribution over the actions ($s_{t,r}x_t$) at each state (S_t, D_t, k, \hat{S}). A strategy π consists of decision rules for each time $\pi = (f_1, f_2,...)$, esp. a stationary decision rule $\pi = (f, f,...)$.

Using these definitions, the optimization of the energy system, with residual load modelled by the Markov Chain $P(D_{t+1}|D_t)$ can be described as a Markov Decision Process⁸. Therefore, the simplifying approximation of a long but finite horizon by an infinite one is applied. The optimal strategy is a solution of the problem

$$\min_{k,\pi} c^{fix} k + \mu_{Sto} \lim_{T \to \infty} \frac{1}{T+1} \mathbb{E} \left[\sum_{t=0}^{T} c^{var} x_t + Voll(D_t + s_t - x_t)^+ \right]$$
(6)

considering $\eta_{sto} = 365 \times 40 \times 24$, the capacity restriction $k \ge x_t \ge 0$, the state transitions a) deterministic: $\hat{S} \ge S_t \ge 0$, and b) stochastic: $P(D_{t+1}|D_t)$ and the initial conditions (S_0, D_0, \hat{S}) . Theorems of existence and structure of an optimal strategy are proved e.g. in [19].

The problem can be decomposed to an output&storage management problem and a capacity optimization. The output problem can be solved separately. This reduces complexity considerably. The solution of the output problem x_t at given capacity is a simple merit order⁹: The technology with lowest variable cost is used as long as its capacity is exhausted and the next more "expensive" technology has to be used and so forth. This optimum output solution makes it possible to define an indirect cost function in which the load *D* that exceeds the non-intermittent capacity $k = (k_1, ..., k_l)$ is valued with the *VoLL*:

$$C(D,k) = \begin{cases} c_1^{var} D & 0 \le D < k_1 \\ \sum_{j=1}^{i} k_j c_j^{var} + c_{i+1}^{var} \left(D - \sum_{j=1}^{i} k_j \right) & k_i \le D < k_{i+1} \\ \sum_{j=1}^{I} k_j c_j^{var} + VoLL \left(D - \sum_{j=1}^{i} k_j \right) & k_I < D \end{cases}$$
(7)

Thus the decision rules simplify to $s_t = f(S_t, D_t | k, \hat{S})$ respectively the strategy to $\pi(k, \hat{S}) = (f, f, ...)$. The decomposition of the problem is then: Determine

$$V(k|S_0, D_0, \hat{S}) = \min_{\pi(k, \overline{S})} \lim_{T \to \infty} \frac{1}{T+1} \mathbb{E}\left[\sum_{t=0}^T C(D_t + s_t|k)\right]$$
(8)

with respect to the deterministic state transition $\hat{S} \ge S_{t+1} = S_t + s_t \ge 0$ and the stochastic $P(D_{t+1}|D_t)$. Determine the capacity k by solving

⁶ For an overview of stochastic analyses in the energy sector see e.g. [12], [16], [17], [21.]

⁷ This optimization problem is therefore decisively more complex, than e.g. the original stochastic optimization problem of the Real Business Cycle Theory. In the latter only storage management has to be optimized without considering the optimization of initial conditions (capacities).

⁸ For an introduction to the topic see e.g. [1], [2], [3], [4] and [5].

⁹ It can be shown that the following rule satisfies the Kuhn-Tucker conditions of the corresponding linear program.

$$\min_{\nu} c^{fix}k + \mu_{Sto}V(k|S_0, D_0, \hat{S})$$
(9)

Both problems can be solved sequentially. With the discrete states $\{1,..., |\hat{S}|\}$ in 10 GWh and $\{1,..., |D|\}$ in 10 GW steps the admissible actions in a state are

$$A((S,D)|k,\hat{S}) = \{s \in \mathbb{N} | 1 \le s + S \le \hat{S}, 1 \le s + D \le \mathbb{1}k\}$$
(10)

Applied to the storage problem [19] proves that from the solution of the linear program:

$$\max_{d((S,D),s)} \sum_{(S,D),s \in A(S,D)} d((S,D),s) C(D+s|k)$$
(11)

$$\sum_{(S,D),s\in A(S,D)} d((S,D),s) = 1$$
(12)

$$d((S,D),s) \ge 0 \tag{13}$$

$$\sum_{s \in A(S,D)} d((S,D),s) = \sum_{S',D',s \in A(S',D')} d((S',D'),s) Prob((S',D'),(S,D),s)$$
(14)

$$Prob((S',D'),(S,D),s) = \begin{cases} P(D'|D) & S' = s + S \\ 0 & otherwise \end{cases}$$
(15)

the decision rule f^{10} can be derived as follows:

$$Prob(s|(S,D)) = \frac{d((S,D),s)}{\sum_{s' \in A(i)} d((S,D),s')}.$$
(16)

Thereby $V(k \mid S_0, D_0, \hat{S})$ can be determined. This allows the solution of the capacity problem with a hillclimbing approach. It proves efficient to include a search direction that is capacity preserving with substitutions of "adjacent"¹¹ technologies. The solution of a case with a storage capacity of 300 GWh is described in detail in the following section

3. Results: Optimal strategy of the stochastic model

We break down the integration of a storage with a capacity of 300 GWh in the electrical system for the stochastic residual load determined in section 2 in scenarios of capacity constraints. These are then gradually relaxed to identify its impact on the optimal strategy, generation capacities, stationary state probabilities and costs. An overview of the scenarios and the results is summarized in Table 4.

Scenario				Strategy	Cost			Index	
	Storage	Capacities	Comment		onategy	Variable	Fixed	Total	Total
1.	0	40,0,10,20,10	Optimized c	apacities	No figure		17250	573082	0.0%
2.	300 GWh	40,0,10,20,10	Opt. stor-	not adjusted capacities	No figure	390171	17250	562671	-1.8%
3.	300 GWh	50,0,0,10,20	age mana-	restrictedly adjusted capacities	Figure 5	377968	18350	561468	-0.2%
4.	300 GWh	50,0,0,20,0	gement	and capacities	Figure 6	378226	17850	556726	-0.9%
								Total	-2.9%

Table 4: Scenarios of storage management and system capacity adjustment

¹⁰ which assigns to each alternative $s \in A(S,D)$ for a given state (S,D) a probability,

 $^{^{\}rm 11}$ - with respect to the fixed costs -

Scenario 1 involves the optimization of non-intermittent generation without storage and capacity constraints. In scenario 2, a 300 GWh storage is introduced, but capacities are not adjusted to the resulting load duration. Differences in the results can be attributed to the optimal storage management. The influence of capacity adjustments, is then examined in two further scenarios: In scenario 3 the capacity structure and storage management is optimized - only restricted by an unchanged total capacity covering peak load and finally this constraint is removed in scenario 4.

In the non-storage reference scenario (1) optimal non-intermittent capacities are optimized to {40,0,10,20,10} GW. Thus total capacity covers peak load of 80 GW and generation equals residual load at any time. As reference total system cost are 573082 mio Euro.

Adding a storage of 300 GWh while maintaining capacities (scenario 2) reduces total cost by 1.8%. The according optimal stationary strategy π is shown in figure 4. Eleven grid lines of the state of charge are plotted in 30-GWh steps on the vertical axis; eight grid lines in steps of 10 GW mark the residual load on the horizontal axis. At each intersection of the grid lines changes in the state of charge are indicated by directed arrows. The length of these arrows corresponds to the quantity of change in 10 GWh steps. A black ring denotes an unchanged state of charge. The stationary probabilities of the Markov decision process with the optimal strategy are visualized at the grid points by the size of the areas of blue circles. To assess the state of the energy system, non-intermittent production and marginal costs of production are shown at each grid point.





Figure 4: Optimal charging strategy (arrows: length represents un/loading in 10 GW steps) and stationary probabilities of the Markov Decision Process (blue circles) – generation capacity adjusted to no storage case; numbers: non-intermittent generation, marginal cost [Eurocents/kWh]

Figure 5: Optimal charging strategy (arrows: length represents un/loading in 10 GW steps) and stationary probabilities of the Markov Decision Process (blue circles) – generation capacity restricted to 80 GW; numbers: non-intermittent generation, marginal cost [Eurocents/kWh]

First capacities are optimized while restricted to 80 GW in total. Thus residual load can be covered by non-intermittent generation under all circumstances $k_R = (50, 0, 0, 10, 20)$ with expected system costs of 561468 million euros.

Storage is used to stabilize residual load longer at higher levels by charging and discharging (Figure 4). This increases utilization of non-intermittent generation capacity and therefore efficiency. For this purpose, the storage is charged below a residual load of 50 GW and discharged above – the stronger, the greater the deviation from 50 GW. Thus a load of 50 GW is achieved over a wide range of the grid, making an expansion of base load capacity profitable. The resulting stationary distribution of states is more unequal among the storage states compared to the perfect foresight case (Figure 6). Thus there is a more waiting with empty (full) storage for higher (lower) residual demand to come.

If the restriction is dropped optimal capacity $k_c = (50, 0, 0, 20, 0)$ sums up to 70 GW with expected system costs of 556726 million euros. Therefore, the residual load of 80 GW cannot be covered without

unloading the storage. If the state of charge is zero, it cannot be discharged any further and load is lost (red rectangle). This state is valued by the social planner with the extreme marginal cost of 500 Eurocents/kWh.

As before storage is used to stabilize residual load (Figure 5). But stabilization is limited by three forms of "stabilization actions" to reduce the risk of losing load: For residual load of

1. 80 GW unloading the storage it is decelerated below a SOC of 150 GWh from 30 to 10 GWh steps. This level of unloading is the minimal level required to avoid lost load. Marginal generation cost rises to 7, which is accepted to avoid lost load besides the state with an SOC of 0.

2. 70 GW below a SOC of 90 GWh the storage is - unlike in the restricted case - not operated at all. Unloading is not necessary to avoid lost load, but marginal cost climb to 7 once again.

3. 60 GW below 50 GWh the storage is loaded – instead of unloaded as in the restricted case - in 10 GWh steps, even if this shifts marginal cost to 7. This operation directly increases the SOC and reduces the risk of being drifted to the right by chance and to end up in the lost load state.

The abdication of unloading above a residual demand of 50 GW and a SOC of 50 GWh reduces the stationary probabilities to almost zero for an SOC of less than 50 GWh. These stabilizing operations form a "buffer zone" in the strategy space. Compared to the perfect foresight case this buffering reduces the efficiency gain of storage.





Figure 6: Optimal charging strategy and stationary probabilities of the Markov Decision Process – unrestricted capacities; numbers: non-intermittent generation, marginal cost [Eurocents/kWh]

Figure 6: Frequencies of state occurrence in the perfect foresight storage model

By increasing the utilization of non-intermittent generation, the load duration curve becomes flatter with a higher base than without storage (without figure). Compared with the perfect foresight model the amount of "Peak Shaving" is not achieved. Therefore, system cost reduction by including a 300 GWh storage is lower under stochastic residual load than under perfect foresight of the same residual load. If the perfect foresight solution is categorized by the frequency of being in a specific state, the corresponding probabilities can be entered in a state diagram (Figure 5). It becomes apparent that the state of charge is distributed more evenly than in the stochastic model (Figure 6).

4. Conclusions

Table 3 shows the system cost and capacity of non-intermittent generation distinct by information scenarios and available storage capacity. Under the perfect foresight hypothesis with the residual load for Germany in 2014, system costs can be reduced by 3.6% using 300 GWh storage capacity. This is achieved by a 10% expansion of the base load capacity and a halving average- and peak load-capacity. If - instead of the residual load data of Germany - 20 simulated time series from the residual load Markov

modelling are used, then the storage option decreases system cost by 4.4% in the perfect foresight case. Again, the base load capacity is increased by 10%, peak load by about 1 GW and mean load is reduced to less than half.

	Without storage	300 GWh storage	Change in system cost		
Perfect foresight	570768	550030	2.6%		
Residual load 2014	{41,13,9,0,13}	{45.6,8.1,4.4,0,5.5}	-3.0%		
Perfect foresight	559010	534196	1 10/		
Markov Load	{40,0,10,20,0}	{44.5,0,5.9,5.6,0.8}	-4.4%		
Stochastic model		561468	2 10/		
Markov Load (80GW)	573082	{50,0,0,10,20}	-2.1%		
Stochastic model	{40,0,10,20,10}	556726	2.0%		
Markov Load		{50.0.0.20.0}	-2.9%		

Table 3: System cost [Mio Euro] and capacities [GW] of non-intermittent generation.

It is shown that under uncertainty at high demand an increasing share of the storage is "frozen" in its charged state to avoid lost load (outages). Therefore, a "buffering" share of the storage is not used actively for the equilibration of load any more. Furthermore, this buffer state of charge is established, if necessary, even in periods of high demand when a fully-charged store would be able to de-stress the system. It can be shown that the size of the buffering area rises as the risk of losing load rises. Thus the efficiency gains of storage decrease as uncertainty in the system rises.

This observation generalizes to other energy carriers like gas: As "generation" capacity falls below peak load, there is a sufficient probability of the latter case to occur and the value of lost load exceeds generation cost by far, optimal storage management includes holding a reserve capacity for peak load. This "buffering" does not occur in the perfect foresight analyses that are still the paradigm of energy systems analysis. Estimates of the potential of storage based on perfect foresight are thus overestimated. Furthermore, the welfare maximizing strategy includes "not unloading" in high marginal cost/price cases. The market implementation of this strategy requires the communalization of lost load costs. We propose a contract solution that includes a premium paid in high load cases for not unloading. This contract makes the storage operator indifferent between reserve holding and unloading. A further option to implement the welfare maximizing strategy would be to operate a sufficiently sized store explicitly as a buffer in the public interest.

Such contracts might be difficult to implement in practice, and so a further option might be the operation of the system with imperfectly adjusted capacities such that non-intermittent generation capacity exceeds peak load. In this case it has to be decided whether the storage is operated "inefficiently" with respect to "full" capacity adjustments, or "efficiently" when peak load capacity is not decommissioned "one for one". The challenges of sustaining rarely-used capacity were a frequent topic at the conference, however.

This analysis refines the assessment of the economic potential of electricity storage, thus contributing to more effective planning of energy systems of the future, where outages are avoided.

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